Module 5: RF Amplifiers

This module focuses on the crucial role of Radio Frequency (RF) Amplifiers in wireless communication systems. We will explore their key characteristics, delve into the specialized design considerations for Low Noise Amplifiers (LNAs) and Power Amplifiers (PAs), and introduce the powerful S-parameter technique for systematic amplifier design.

5.1 Characteristics of RF Amplifiers

RF amplifiers are fundamental building blocks in almost every wireless system, from your mobile phone to satellite communication. They are responsible for boosting the power or voltage of a weak RF signal to a usable level. However, unlike audio amplifiers, RF amplifiers must operate reliably at very high frequencies, where parasitic effects become significant and maintaining signal integrity is challenging. Several key characteristics define the performance of an RF amplifier:

Gain:

- **Definition: Gain is the primary characteristic of an amplifier, representing its ability to increase the power or voltage of a signal. It is typically expressed as a ratio (output to input) or in decibels (dB).**
- **Formulas:**
  - **Power Gain (Gp): The ratio of output power to input power.**
    Gp=PinPout (unitless ratio)
    In decibels (dB):
    Gp(dB)=10log10(PinPout)
  - **Voltage Gain (Gv): The ratio of output voltage to input voltage.**
    Gv=VinVout (unitless ratio)
    In decibels (dB) for matched impedances (Rin=Rout):
    Gv(dB)=20log10(VinVout)
  - **Transducer Power Gain (GT): This is the most practical and useful gain definition in RF amplifier design. It considers the impedance mismatch at both the input and output of the amplifier. It's defined as the power delivered to the load divided by the maximum available power from the source.**
    GT=Pavailable,sourcePload
    This gain can be calculated using S-parameters, which we will discuss later.
- **Importance: A high gain is desirable for weak signals (e.g., in receivers) to bring them above the noise floor. However, excessive gain can lead to instability (oscillation).**

Numerical Example: Gain Calculation

An RF amplifier receives an input power of 10 microWatts (10 μW) and delivers an output power of 50 milliWatts (50 mW).

Pin=10×10−6 W

Pout=50×10−3 W

Power Gain (ratio):

Gp=10×10−650×10−3=1050000=5000

Power Gain (dB):

Gp(dB)=10log10(5000)=10×3.699=36.99 dB

Noise Figure (NF):

- Definition: All electronic components generate some level of random electrical noise. The Noise Figure (or Noise Factor) quantifies how much an amplifier degrades the Signal-to-Noise Ratio (SNR) of a signal as it passes through. A perfect (ideal) amplifier would add no noise and have a Noise Figure of 1 (or 0 dB). Real amplifiers always have NF > 1 (or > 0 dB).
- Formula:
NF=SNRoutSNRin (unitless ratio, where SNR is signal power / noise power)
In decibels (dB):
NF(dB)=10log10(NF)
- Importance: A low Noise Figure is crucial for the very first amplifier stage in a receiver chain, typically called the Low Noise Amplifier (LNA). Any noise added at this stage is amplified by all subsequent stages, severely impacting the overall receiver sensitivity. Higher gain in later stages cannot compensate for poor SNR at the input.
- Friis' Formula for Cascaded Noise Figure: When multiple stages are cascaded, the overall noise figure is dominated by the noise figure and gain of the first stage.
NFtotal=NF1+G1NF2−1+G1G2NF3−1+…
(Where NFn and Gn are the noise factor and power gain of the nth stage, respectively, in linear units, not dB). This formula clearly shows why the first stage (LNA) must have a very low NF and reasonably high gain.

Numerical Example: Noise Figure Calculation

An LNA has an input SNR of 20 dB and an output SNR of 17 dB.

$SNR_{in}(dB)=20\ dB \Rightarrow SNR_{in}=10^{(20/10)}=100$

$SNR_{out}(dB)=17\ dB \Rightarrow SNR_{out}=10^{(17/10)}=50.11$

Noise Figure (ratio):

$NF=\frac{SNR_{out}}{SNR_{in}}=\frac{50.11}{100}\approx 1.995$

Noise Figure (dB):

$NF(dB)=10\log_{10}(1.995)\approx 3.0\ dB$

Linearity:

- **Definition: Linearity describes how faithfully an amplifier reproduces the input signal without introducing distortion. A perfectly linear amplifier would only amplify the input signal, producing an output that is a scaled version of the input. In reality, all amplifiers exhibit some non-linearity, especially at higher power levels.**
- **Effects of Non-linearity:**
    - **Harmonic Distortion: Generation of output signals at integer multiples of the input frequency.**
    - **Intermodulation Distortion (IMD): When multiple input frequencies are present, non-linearity creates new frequencies (intermodulation products) that are combinations (sums and differences) of the input frequencies and their harmonics. These can fall within the desired signal band and cause interference.**
    - **Gain Compression: As input power increases, the amplifier's gain starts to decrease.**
- **Key Metrics for Linearity:**
    - **1-dB Compression Point (P1dB): The output power level at which the amplifier's gain has dropped by 1 dB compared to its linear (small-signal) gain. It's a measure of the maximum usable output power for relatively linear operation.**
    - **Third-Order Intercept Point (IP3 or IIP3/OIP3): A hypothetical point where the power of the third-order intermodulation products would equal the power of the fundamental (desired) signals if the amplifier remained linear. It's an extrapolated value, usually significantly higher than P1dB. A higher IP3 indicates better linearity. IIP3 is input-referred, OIP3 is output-referred.**
- **Importance: Linearity is critical in communication systems where multiple signals are present (e.g., cellular base stations, broadband radios) to prevent interference between channels. It's also important for maintaining signal quality in modulated signals.**

Numerical Example: 1-dB Compression Point

An amplifier has a small-signal gain of 20 dB. If its output power at 1-dB compression is 1 Watt (1000 mW), what is its P1dB?

Output P1dB = 1 W.

Small-signal gain = 20 dB.

At the 1-dB compression point, the actual gain is 20 dB - 1 dB = 19 dB.

Input power at P1dB (Pin,P1dB):

19=10log10(Pout,P1dB/Pin,P1dB)

1.9=log10(1000 mW/Pin,P1dB)

101.9=1000/Pin,P1dB

79.43=1000/Pin,P1dB

Pin,P1dB=1000/79.43≈12.59 mW

Power Added Efficiency (PAE):

- **Definition: PAE is a crucial metric for Power Amplifiers (PAs). It measures how efficiently a PA converts DC power (from its power supply) into RF output power, while also accounting for the RF input power. It's a better metric than simple drain/collector efficiency because it considers the power gain of the amplifier.**
- **Formula:**
  PAE=PDC,suppliedPRF,out−PRF,in×100%
  Where PRF,out is the RF output power, PRF,in is the RF input power, and PDC,supplied is the DC power drawn from the supply.
- **Importance: High PAE is vital in battery-powered devices (mobile phones, IoT devices) to extend battery life and in high-power applications (base stations) to reduce energy consumption and heat dissipation. A higher PAE means less waste heat, which simplifies cooling requirements.**

Numerical Example: PAE Calculation

A power amplifier has an RF input power of 10 dBm, an RF output power of 30 dBm, and draws 200 mA from a 5V DC supply.

Convert powers to Watts:

PRF,in=10(10/10) mW=10 mW=0.01 W

PRF,out=10(30/10) mW=1000 mW=1 W

Calculate DC power supplied:

PDC,supplied=VDC×IDC=5 V×0.200 A=1 W

Calculate PAE:

PAE=PDC,suppliedPRF,out−PRF,in×100%

PAE=1 W1 W−0.01 W×100%=10.99×100%=99%

This is an exceptionally high PAE, typical of highly optimized modern PAs, but demonstrates the calculation.

Bandwidth:

- **Definition: The bandwidth of an RF amplifier refers to the range of frequencies over which it performs acceptably well (e.g., its gain is within a specified range, or its other characteristics meet certain criteria).**
- **Metrics:**
    - **3-dB Bandwidth: The range of frequencies between the upper and lower 3-dB points, where the gain has dropped by 3 dB (half power) from its maximum value.**
    - **Operating Bandwidth: The specific frequency range where the amplifier is designed to operate.**
- **Importance: Different applications require different bandwidths. A narrow-band amplifier (e.g., for a specific radio channel) will have high gain and efficiency over a small frequency range, while a wide-band amplifier (e.g., for an oscilloscope or general-purpose test equipment) sacrifices some gain and efficiency for operation over a broad spectrum.**

5.2 Low Noise Amplifiers (LNAs)

Low Noise Amplifiers (LNAs) are the very first active components in a receiver chain, directly following the antenna. Their primary purpose is to amplify the extremely weak RF signals captured by the antenna without adding significant noise of their own. The performance of an LNA largely dictates the overall sensitivity of the entire receiver system.

Design considerations for LNAs:

1. **Noise Matching (Most Critical):**
    - **Concept: This is the paramount consideration for an LNA. To achieve the lowest possible noise figure, the LNA's input impedance must be matched to the *optimum noise impedance* of the active device (transistor), not necessarily to the source impedance for maximum power transfer (e.g., 50 Ohms). These**

two matching conditions (for minimum noise and maximum power) are often different.
  - Trade-off: There's usually a trade-off between achieving minimum noise figure and achieving maximum power gain. Often, the design aims for a compromise: a low noise figure (close to the device's minimum noise figure, Fmin) with acceptable gain, rather than strictly maximum gain.
  - Tools: Noise circles on the Smith Chart (discussed in Section 5.4) are used to visualize and design for noise matching.
2. Power Gain (Gp):
  - Concept: While noise is paramount, the LNA still needs to provide sufficient gain to raise the signal level above the noise floor of subsequent stages. A typical LNA gain might be 15-25 dB.
  - Impact on overall NF: As shown by Friis' formula, the gain of the first stage (LNA) significantly desensitizes the overall receiver to noise contributed by later stages. A higher LNA gain means the noise from later stages has less impact on the overall system NF.
  - Trade-off: Too much gain in the LNA can lead to instability or force the LNA into compression at moderate input signal levels, which is undesirable.
3. Stability:
  - Concept: An amplifier is stable if it does not oscillate (produce unwanted output signals on its own) under any source and load impedance conditions. RF amplifiers, especially those with high gain, are prone to instability due to parasitic feedback paths.
  - Unconditional Stability: An amplifier is unconditionally stable if it will not oscillate for any passive source and load impedance. This is often a design goal for LNAs.
  - Conditional Stability: An amplifier is conditionally stable if it will oscillate only for specific combinations of source and load impedances. In this case, the design must ensure that the actual operating source and load impedances fall within the stable region.
  - Metrics & Tools: Stability circles (discussed in Section 5.4) are used on the Smith Chart to analyze and ensure stability. The Linvill stability factor (C) and Rollett stability factor (K) are used for quantitative assessment. For unconditional stability, $K>1$ and $|\Delta|<1$, where $\Delta=S_{11}S_{22}-S_{12}S_{21}$.
4. Linearity:
  - Concept: While LNAs handle very small input signals, they can still be driven into non-linearity by strong interfering signals (blockers) that are outside the desired band. Non-linearity can cause intermodulation products that fall into the desired band, effectively raising the noise floor or corrupting the signal.

- **Importance: A good LNA should have sufficient linearity (e.g., a reasonable IIP3) to handle strong out-of-band signals without generating in-band interference.**

5. **Input/Output Matching:**
    - **Concept: After noise matching the input, the output of the LNA needs to be matched to the input of the next stage (often 50 Ohms) for efficient power transfer.**
    - **Tools: Constant gain circles and stability circles are used.**

Design examples of common LNA configurations:

LNAs are typically built using Field-Effect Transistors (FETs), especially MOSFETs or HEMTs (High Electron Mobility Transistors), due to their inherently low noise characteristics at high frequencies. Bipolar Junction Transistors (BJTs) can also be used. Here, we'll look at two common single-stage configurations.

- **Common Source (or Common Emitter for BJT) LNA:**
    - **Description: This is a very common configuration for LNAs due to its high gain. The input signal is applied to the gate (or base), and the output is taken from the drain (or collector). The source (or emitter) is typically grounded or bypassed to ground for RF.**
    - **Advantages: Provides significant voltage and power gain. Relatively easy to implement input and output matching networks. Offers good isolation between input and output if the reverse isolation (S12) of the transistor is low.**
    - **Noise Matching: The input matching network is crucial. It transforms the source impedance (e.g., 50 Ohms from the antenna) to the optimum noise impedance (Zopt) of the transistor. This network often uses inductors and capacitors.**
    - **Stability: Can be prone to instability, especially at high frequencies, due to internal feedback capacitance (Cgd or Cbc). Neutralization or resistive loading might be required to ensure stability.**
    - **Typical Example (conceptual): A transistor (e.g., MOSFET) with its gate connected to an input matching network (e.g., an L-section LC network) that matches the antenna to Zopt. The drain is connected to an output matching network that matches to the next stage (e.g., 50 Ohm filter). The source is typically grounded. Biasing resistors and decoupling capacitors are also present.**
- **Common Gate (or Common Base for BJT) LNA:**
    - **Description: In this configuration, the input signal is applied to the source (or emitter), and the output is taken from the drain (or**

collector). The gate (or base) is typically grounded or bypassed for RF.

- Advantages:
    - Excellent Stability: Inherently more stable than common-source due to reduced internal feedback (the gate is grounded, reducing the effect of Cgd). This makes it suitable for higher frequencies.
    - Wideband Potential: Can offer wider bandwidths compared to common-source.
    - Good Input Match: The input impedance of a common-gate stage is typically around 1/gm (where gm is transconductance), which can be designed to be close to 50 Ohms, simplifying input matching.
- Disadvantages: Generally offers lower power gain compared to common-source configurations. The noise figure might be slightly higher than an optimized common-source LNA at lower frequencies, but its stability and input match advantages can make it preferable at higher frequencies.
- Typical Example (conceptual): A transistor with its source connected to the input matching network. The gate is connected to ground via a capacitor for RF. The drain is connected to an output matching network. Biasing components are added.

Numerical Example: LNA Gain and Noise

A receiver chain consists of an LNA, a mixer, and an IF amplifier.

LNA: Gain (G1) = 20 dB, Noise Figure (NF1) = 1.5 dB

Mixer: Gain (G2) = -6 dB (it's a lossy component), Noise Figure (NF2) = 8 dB

IF Amplifier: Gain (G3) = 30 dB, Noise Figure (NF3) = 4 dB

We need to convert gains and noise figures to linear ratios for Friis' formula.

G1=10(20/10)=100

NF1=10(1.5/10)=1.413

G2=10(−6/10)=0.251

NF2=10(8/10)=6.31

G3=10(30/10)=1000

NF3=10(4/10)=2.512

Calculate overall noise figure using Friis' formula:

NFtotal=NF1+G1NF2−1+G1G2NF3−1

NFtotal=1.413+1006.31−1+100×0.2512.512−1

NFtotal=1.413+1005.31+25.11.512

NFtotal=1.413+0.0531+0.0602

NFtotal=1.5263

Overall Noise Figure in dB:

NFtotal(dB)=10log10(1.5263)≈1.83 dB

This example clearly shows how the LNA's noise figure (NF1=1.5 dB) dominates the overall system noise figure (1.83 dB), while the noise from the mixer and IF amplifier contribute relatively little due to the LNA's gain.

5.3 Power Amplifiers (PAs)

Power Amplifiers (PAs) are designed to deliver high output power to a load, typically an antenna, with high efficiency. They are the final active stage in a transmitter chain. Unlike LNAs, where noise and small-signal linearity are paramount, PAs prioritize output power, efficiency, and robustness, often at the expense of linearity.

Classes of power amplifiers:

Power amplifier classes categorize amplifiers based on their conduction angle (the portion of the input signal cycle during which the active device conducts current) and the operating point of the transistor. Different classes offer trade-offs between efficiency, linearity, and complexity.

- Class A:
  - Conduction Angle: 360 degrees (conducts throughout the entire input cycle).
  - Operation: The transistor is biased in its linear region such that current flows constantly.
  - Advantages: Excellent linearity, minimal distortion. Simple design.
  - Disadvantages: Very low efficiency (theoretically maximum 50% for resistive load, 25% for transformered load). Significant quiescent (idle) current draw, even with no input signal, leading to high power dissipation and heat.

    - **Applications: High-fidelity audio, small-signal lab amplifiers, drivers for more efficient PA stages where linearity is critical. Not commonly used as high-power RF PAs due to poor efficiency.**
- **Class B:**
    - **Conduction Angle: 180 degrees (conducts for half of the input cycle).**
    - **Operation: The transistor is biased at cutoff, so it only conducts when the input signal exceeds a certain threshold. Typically, two transistors are used in a push-pull configuration, with one handling the positive half-cycle and the other handling the negative half-cycle.**
    - **Advantages: Higher efficiency than Class A (theoretically maximum 78.5%). Lower quiescent power dissipation.**
    - **Disadvantages: Significant crossover distortion at the zero-crossing point, where the signal transitions from one transistor to the other. Poor linearity.**
    - **Applications: Audio power amplifiers where some distortion is tolerable or compensated for. Not ideal for RF without further measures.**
- **Class AB:**
    - **Conduction Angle: Between 180 and 360 degrees (slightly more than half a cycle).**
    - **Operation: A compromise between Class A and Class B. Each transistor in a push-pull pair is biased slightly above cutoff, just enough to eliminate crossover distortion.**
    - **Advantages: Good balance of linearity and efficiency (higher than Class A, lower than Class B, but with less distortion).**
    - **Disadvantages: Moderate efficiency (typically 50-70%).**
    - **Applications: Very common for general-purpose linear RF power amplifiers, especially in communication systems where both linearity and reasonable efficiency are required (e.g., cellular base stations, OFDM transmitters).**
- **Class C:**
    - **Conduction Angle: Less than 180 degrees. The transistor conducts for a small portion of the input cycle.**
    - **Operation: Biased well below cutoff. Requires a tuned (resonant) output circuit (e.g., LC tank) to filter the pulsed output current and reconstruct the sinusoidal waveform at the desired frequency.**
    - **Advantages: Very high efficiency (theoretically up to 100%, practically 80-90%).**
    - **Disadvantages: Highly non-linear; significant distortion unless a tuned output circuit is used. Cannot amplify amplitude-modulated (AM) signals linearly.**

    - **Applications: Used in applications where efficiency is paramount and linearity is not a primary concern, or where the signal is constant amplitude (e.g., FM transmitters, CW (continuous wave) transmitters).**
- **Switching Classes (Class D, E, F):**
    - **Class D:**
        - **Operation: The transistor operates as a switch, rapidly switching between saturation (fully ON) and cutoff (fully OFF). The input signal is typically converted to a pulse-width modulated (PWM) signal. A low-pass filter at the output reconstructs the amplified analog waveform.**
        - **Advantages: Very high theoretical efficiency (up to 100%), as the transistor spends little time in the dissipative active region.**
        - **Disadvantages: Requires complex control circuitry (PWM modulator, demodulator). Primarily used at lower RF frequencies or for audio.**
    - **Class E/F (and others like G, H):**
        - **Operation: These are highly efficient RF power amplifier classes that use harmonic-tuned output networks. They shape the voltage and current waveforms at the transistor's output to minimize the overlap where both voltage and current are simultaneously high (which causes power dissipation). This is achieved by using reactive components that present specific impedances at the fundamental and harmonic frequencies.**
        - **Advantages: Extremely high efficiency (often over 85-90% for Class E/F).**
        - **Disadvantages: Very sensitive to load impedance variations and frequency changes. Complex output matching network design.**
        - **Applications: Highly desirable for battery-powered devices (mobile phones, Wi-Fi, Bluetooth) and high-power base stations where efficiency is critical to save power and reduce cooling costs.**

Efficiency, linearity, and heat dissipation considerations:

These three characteristics are tightly interconnected and represent the primary trade-offs in PA design.

- **Efficiency:**

- **Importance: Direct impact on battery life in portable devices and operating costs/cooling requirements in fixed installations. Higher efficiency means less wasted power as heat.**
    - **Trade-off with Linearity: Generally, classes with higher efficiency (Class C, D, E, F) achieve this by operating the transistor in a non-linear switching mode, thus sacrificing linearity. Classes designed for high linearity (Class A, AB) typically have lower efficiency because the transistor spends more time in the dissipative active region.**
    - **Metrics: Drain Efficiency (for FETs) or Collector Efficiency (for BJTs), and Power Added Efficiency (PAE).**
- **Linearity:**
    - **Importance: Crucial for complex modulation schemes (e.g., QAM, OFDM) used in modern wireless communications. Non-linear distortion can cause spectral regrowth (spreading of the signal into adjacent channels) and intermodulation products, leading to interference and reduced data rates.**
    - **Trade-off with Efficiency: Improving linearity often means moving towards Class A or AB operation, which reduces efficiency. Techniques like pre-distortion and feedback loops are used to linearize efficient PAs, but they add complexity and cost.**
    - **Metrics: 1-dB Compression Point (P1dB), Third-Order Intercept Point (IP3).**
- **Heat Dissipation:**
    - **Importance: All power not converted to useful RF output is dissipated as heat within the transistor and associated components. Excessive heat reduces device lifetime, degrades performance, and can lead to thermal runaway or catastrophic failure.**
    - **Design Considerations:**
        - **Heat Sinking: Designing adequate heat sinks to transfer heat away from the transistor package.**
        - **Thermal Management: Considering the thermal resistance of the package, board, and heat sink.**
        - **Packaging: Using packages that facilitate efficient heat removal (e.g., flange-mount packages).**
        - **Bias Point Stability: Ensuring the operating point doesn't drift with temperature.**
    - **Relationship to Efficiency: A PA with 50% efficiency delivering 10 Watts of RF output power will dissipate 10 Watts of heat (50% of total 20W DC input). A PA with 80% efficiency delivering 10 Watts RF will only dissipate 2.5 Watts of heat (20% of total 12.5W DC input). Clearly, higher efficiency significantly reduces cooling requirements.**

Design challenges and trade-offs in PAs:

Designing effective RF PAs involves navigating a complex set of conflicting requirements and making strategic trade-offs:

1. **Efficiency vs. Linearity: This is the most fundamental trade-off. Highly linear amplifiers (Class A, AB) are inefficient. Highly efficient amplifiers (Class C, E, F) are inherently non-linear. Modern communication standards (like 5G, Wi-Fi) demand both high data rates (requiring high linearity for complex modulation) and long battery life/energy savings (requiring high efficiency). This pushes for advanced linearization techniques and envelope-tracking power supplies.**
2. **Output Power vs. Breakdown Voltage/Current: Selecting the right transistor involves ensuring it can handle the required output power without exceeding its breakdown voltage or current limits. Higher power often means larger devices, which can introduce more parasitic capacitance and inductance, limiting high-frequency performance.**
3. **Gain vs. Stability: PAs often require high gain to boost signals to high power levels. High gain makes the amplifier more susceptible to instability (oscillation). Careful design of input/output matching networks and feedback paths is necessary to ensure stability.**
4. **Impedance Matching for High Power:**
    - **The optimal load impedance for maximum power and efficiency (often called the "load-pull" impedance) is typically very low (e.g., a few Ohms or fractions of an Ohm) for high-power transistors. Matching this very low impedance to a standard 50 Ohm system impedance (like an antenna) requires very high Q (Quality Factor) matching networks.**
    - **These high Q networks can be lossy themselves, reducing overall efficiency, and have narrow bandwidths.**
    - **The matching networks must also handle high currents and voltages without breaking down.**
5. **Thermal Management: As discussed, heat dissipation is a major challenge. Ensuring proper heat sinking, thermal conductivity of materials, and often active cooling (fans, liquid cooling for very high power) is essential for reliability and longevity.**
6. **Bandwidth vs. Efficiency/Complexity: Achieving high efficiency in switching classes (E, F) often relies on precise harmonic tuning, which intrinsically limits the amplifier's bandwidth. Wideband PAs are generally less efficient and more complex to design.**
7. **Cost and Size: High-power transistors, specialized matching components, and sophisticated thermal management add to the cost and physical size of the PA module.**

8. **Reliability and Robustness: PAs must be robust enough to withstand load mismatches (e.g., a faulty antenna or short/open circuit) without catastrophic failure. Protection circuitry is often integrated.**

5.4 Amplifier Design using S-parameters

Designing RF amplifiers, especially at microwave frequencies, becomes incredibly complex using traditional lumped-element analysis (voltage, current, impedance) due to parasitic effects and the distributed nature of components. S-parameters (Scattering Parameters) provide a powerful and practical alternative.

What are S-parameters?

S-parameters characterize the behavior of an N-port network (like a transistor or an amplifier) in terms of incident and reflected power waves at its ports. Unlike Z or Y parameters, which are based on open/short circuits (difficult to achieve at RF), S-parameters are measured using matched 50 Ohm (or other characteristic impedance) terminations, which is practical at RF.

For a 2-port device (like a transistor with input and output ports):

- **S11: Input reflection coefficient with the output terminated in Z0.**
  - **S11=b1/a1 (when a2=0)**
- **S21: Forward transmission coefficient (forward gain) with the output terminated in Z0.**
  - **S21=b2/a1 (when a2=0)**
- **S12: Reverse transmission coefficient (reverse isolation) with the input terminated in Z0.**
  - **S12=b1/a2 (when a1=0)**
- **S22: Output reflection coefficient with the input terminated in Z0.**
  - **S22=b2/a2 (when a1=0)**

Where an represents the complex amplitude of the incident voltage wave at port n, and bn represents the complex amplitude of the reflected voltage wave at port n.

S-parameters are frequency-dependent and are typically provided by semiconductor manufacturers in data sheets for RF transistors.

Amplifier Design Process using S-parameters:

The design process generally involves several steps to achieve desired gain, noise, and stability, often using the Smith Chart as a graphical aid for matching networks.

1. **Stability Analysis (Stability Circles):**
    - **Concept: Before attempting to design for gain or noise, an amplifier must be stable. S-parameters allow calculation of stability circles on the Smith Chart. These circles define regions of source or load impedance that would cause the amplifier to become unstable (oscillate).**
    - **Input Stability Circle: Defines regions in the source impedance (ΓS) plane that lead to instability.**
    - **Output Stability Circle: Defines regions in the load impedance (ΓL) plane that lead to instability.**
    - **Formulas for Centers (CS,CL) and Radii (rS,rL) of Stability Circles: The derivation is complex, but the key is that these are calculated from $S11$,$S12$,$S21$,$S22$ and the determinant $\Delta = S11S22 - S12S21$.**
      $CS = |S11|2 - |\Delta|2(S11 - \Delta S22*)*$
      $rS = ||S11|2 - |\Delta|2||S12S21|$
      $CL = |S22|2 - |\Delta|2(S22 - \Delta S11*)*$
      $rL = ||S22|2 - |\Delta|2||S12S21|$
      (The asterisk * denotes complex conjugate).
    - **Unconditional Stability Criteria (K-Δ Test): For an amplifier to be unconditionally stable (i.e., stable for any passive source and load impedance), two conditions must be met:**
        - $K = 2|S12S21|1 - |S11|2 - |S22|2 + |\Delta|2 > 1$
        - $|\Delta| < 1$
          If these conditions are met, the stability circles lie entirely outside the Smith Chart, or they enclose the center but the stable region is outside the circle, meaning all points on the Smith Chart are stable. If not, conditional stability exists, and unstable regions must be avoided.
2. **Constant Gain Circles:**
    - **Concept: These are circles on the Smith Chart that represent all possible source or load reflection coefficients (ΓS or ΓL) that yield a specific value of transducer power gain (GT).**
    - **Purpose: Used to design input and output matching networks to achieve a desired gain level.**
    - **Formulas: The equations for the centers and radii of these circles are also derived from S-parameters and the desired gain. The maximum available gain ($GMAX$) occurs when the input and output are simultaneously conjugate matched.**
3. **Noise Circles:**
    - **Concept: For LNA design, these circles are plotted on the Smith Chart (specifically, the ΓS plane). Each circle represents all source reflection coefficients (ΓS) that result in a specific noise figure.**
    - **Purpose: To identify the optimum source reflection coefficient (Γopt) that yields the minimum noise figure ($Fmin$) for the**

transistor. Other circles show where the noise figure increases by a certain amount (e.g., 0.5 dB, 1 dB) above F_{min}.
    - Parameters Required: F_{min} (minimum noise figure), Γ_{opt} (optimum source reflection coefficient for F_{min}), and R_n (equivalent noise resistance of the device). These are typically provided in transistor datasheets.
    - Design Approach: For LNAs, the input matching network is designed to transform the source impedance (e.g., 50 Ohm antenna) to a Γ_S that is close to Γ_{opt}, balancing low noise with acceptable gain and stability.

Simultaneous Conjugate Matching:

- Concept: This is a technique used to achieve maximum power transfer to and from an amplifier, especially useful when the transistor is unconditionally stable and the primary goal is maximum gain (e.g., in intermediate amplifier stages, not necessarily LNAs or PAs where noise or efficiency might dictate different matching).
- Conditions: For maximum power transfer, the following conditions must be met:
    1. The source reflection coefficient (Γ_S) must be the complex conjugate of the input reflection coefficient of the transistor (S_{11} when the output is matched, or more generally, the input reflection coefficient looking into the transistor with a specific load, Γ_{in}).
    $\Gamma_S = \Gamma_{in}^*$
    2. The load reflection coefficient (Γ_L) must be the complex conjugate of the output reflection coefficient of the transistor (S_{22} when the input is matched, or more generally, the output reflection coefficient looking out of the transistor with a specific source, Γ_{out}).
    $\Gamma_L = \Gamma_{out}^*$
- Transducer Power Gain (G_T): When simultaneously conjugate matched, the transducer power gain (G_T) is maximized. The formula for G_T is:
$$G_T = \frac{P_{load}}{P_{available,source}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1 - S_{11}\Gamma_S - S_{22}\Gamma_L + S_{11}S_{22}\Gamma_S\Gamma_L - S_{12}S_{21}\Gamma_S\Gamma_L|^2}$$
This simplifies significantly when Γ_S and Γ_L are chosen for simultaneous conjugate match.
- Input and Output Matching Networks: Separate input and output matching networks are designed using components (inductors, capacitors, transmission line sections) to transform the source and load impedances to Γ_S and Γ_L respectively. This is done graphically on the Smith Chart.

Design Steps (General Amplifier using S-parameters):

1. Select Transistor: Choose a transistor based on frequency, power level, noise requirements, and gain. Obtain its S-parameters (and noise parameters if LNA) at the desired operating frequency and bias point.
2. Stability Analysis:
    - Calculate K and Δ.
    - Plot stability circles on the Smith Chart. Identify stable and unstable regions for ΓS and ΓL. Ensure the design avoids unstable regions.
    - If unconditionally unstable, consider feedback or resistive loading to stabilize.
3. Gain Design (for general amplifier) or Noise Design (for LNA):
    - For general amplifier: Plot constant gain circles to identify regions of desired gain. Aim for the circle that gives the required gain while maintaining stability. If maximum gain is desired and stable, pursue simultaneous conjugate match.
    - For LNA: Plot noise circles. Identify Γopt for minimum noise. Choose a ΓS close to Γopt that provides an acceptable noise figure and also ensures stability and reasonable gain.
4. Input Matching Network Design:
    - Based on the chosen ΓS (from stability/gain/noise considerations), design an input matching network that transforms the actual source impedance (e.g., 50 Ohms) to the required ΓS. This involves moving on the Smith Chart from the source impedance to ΓS using reactive components or transmission lines.
5. Output Matching Network Design:
    - Based on the chosen ΓL (from stability/gain considerations), design an output matching network that transforms the required load impedance (e.g., 50 Ohms) to ΓL. This involves moving on the Smith Chart from the load impedance to ΓL using reactive components or transmission lines.
6. Bias Network Design: Design a stable DC biasing circuit for the transistor to set its operating point correctly. This network must be transparent at RF (e.g., using RF chokes and bypass capacitors).
7. Simulation and Optimization: Use RF circuit simulation software (e.g., Keysight ADS, Cadence AWR, Ansys HFSS, QucsStudio) to verify performance, optimize component values, and refine the design, accounting for parasitic elements.
8. Layout and Fabrication: Implement the design on a PCB, paying careful attention to trace widths for impedance control, component placement, and thermal management.

Numerical Example: Stability and Gain (Conceptual on Smith Chart)

Let's say we have a transistor at 2 GHz with the following S-parameters:

$S11=0.5\angle 160^{\circ}$

$S21=3.0\angle 45^{\circ}$

$S12=0.05\angle -10^{\circ}$

$S22=0.6\angle -80^{\circ}$

1. **Check K-Δ Stability:**
   $\Delta=S11S22-S12S21$
   $\Delta=(0.5\angle 160^{\circ})(0.6\angle -80^{\circ})-(0.05\angle -10^{\circ})(3.0\angle 45^{\circ})$
   $\Delta=(0.5\times 0.6)\angle(160-80)^{\circ}-(0.05\times 3.0)\angle(-10+45)^{\circ}$
   $\Delta=0.3\angle 80^{\circ}-0.15\angle 35^{\circ}$
   Convert to rectangular:
   $0.3(\cos 80^{\circ}+j\sin 80^{\circ})=0.3(0.1736+j0.9848)=0.05208+j0.29544$
   $0.15(\cos 35^{\circ}+j\sin 35^{\circ})=0.15(0.8192+j0.5736)=0.12288+j0.08604$
   $\Delta=(0.05208-0.12288)+j(0.29544-0.08604)=-0.0708+j0.2094$
   $|\Delta|=(-0.0708)2+(0.2094)2=0.00501+0.04385=0.04886\approx 0.221$
   Since $|\Delta|=0.221<1$, the first condition for unconditional stability is met.
   Now, calculate K:
   $K=2|S12S21|1-|S11|2-|S22|2+|\Delta|2$
   $|S11|2=(0.5)2=0.25$
   $|S22|2=(0.6)2=0.36$
   $|S12S21|=|(0.05)(3.0)|=0.15$
   $K=2\times 0.151-0.25-0.36+(0.221)2=0.31-0.25-0.36+0.0488$
   $K=0.30.4388\approx 1.463$
   Since $K=1.463>1$, the second condition is also met.
   Conclusion: This transistor is unconditionally stable at 2 GHz for this bias point. This means we can choose any passive source and load impedance without causing oscillation.
2. **Designing for Maximum Gain (Simultaneous Conjugate Match):**
   Since the transistor is unconditionally stable, we can achieve maximum transducer power gain (GMAX) by performing simultaneous conjugate matching. This involves finding the source and load reflection coefficients that conjugate match the device's input and output.
   The formulas for the source and load reflection coefficients for simultaneous conjugate match (ΓMS and ΓML) are:
   $\Gamma MS=2C1B1\pm B12-4|C1|2$
   $\Gamma ML=2C2B2\pm B22-4|C2|2$
   Where:
   $C1=S11-\Delta S22*$

$B_1 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\Delta|^2$
$C_2 = S_{22} - \Delta S_{11}^*$
$B_2 = 1 - |S_{22}|^2 + |S_{11}|^2 - |\Delta|^2$
Calculating these values would involve more complex arithmetic, but the principle is:

- **You calculate Γ_{MS} and Γ_{ML} using the S-parameters.**
- **On the Smith Chart, you then plot Γ_{MS}^* (the input impedance that the transistor "wants" to see) and Γ_{ML}^* (the output impedance that the transistor "wants" to see).**
- **You then design an input matching network to transform your source (e.g., 50 Ohms, or $\Gamma_S = 0$) to Γ_{MS}^*.**
- **You design an output matching network to transform your load (e.g., 50 Ohms, or $\Gamma_L = 0$) to Γ_{ML}^*.**

3. **For this specific transistor, if we were to calculate these, let's assume we find (for illustration):**
   $\Gamma_{MS} = 0.7\angle -150^\circ$
   $\Gamma_{ML} = 0.8\angle 90^\circ$
   This means:
   - **Your input matching network needs to transform your 50 Ohm source to an impedance that corresponds to $\Gamma_S = 0.7\angle -150^\circ$ on the Smith Chart.**
   - **Your output matching network needs to transform your 50 Ohm load to an impedance that corresponds to $\Gamma_L = 0.8\angle 90^\circ$ on the Smith Chart.**
4. **The actual process of designing these matching networks involves graphically moving points on the Smith Chart using series or shunt lumped components (L/C) or sections of transmission line, as described in Module 2.5.**

S-parameters and the Smith Chart together form a powerful and indispensable toolkit for modern RF amplifier design, allowing engineers to systematically design for performance metrics like gain, noise, and stability.